

Academic Standards for Mathematics

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Pennsylvania Department of Education

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V. INTRODUCTION

This document includes Mathematics Standards:

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The Mathematics Standards describe what students should know and be able to do at four grade levels (third, fifth, eighth and eleventh). They reflect the increasing complexity and sophistication that students are expected to achieve as they progress through school.

With each Standard divided into conceptual strands, this document avoids repetition of learned skills and makes an obvious progression across grade levels less explicit. Teachers shall expect that students know and can apply the concepts and skills expressed at the preceding level. Consequently, previous learning is reinforced but not re-taught.

Students who achieve these mathematical standards will be able to communicate mathematically. Although it is an interesting and enjoyable study for its own sake, mathematics is most appropriately used as a tool to help organize and understand information from other academic disciplines. Because our capacity to deal with all things mathematical is changing rapidly, students must be able to bring the most modern and effective technology to bear on their learning of mathematical concepts and skills.

A glossary is included to assist the reader in understanding terminology contained in the standards. Words in bold faced text are included in the glossary.

2.1. Numbers, Number Systems and Number Relationships

Strand	2.1.3. GRADE 3	2.1.5. GRADE 5	2.1.8. GRADE 8	2.1.11. GRADE 11
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>				
A. Count and compare numbers	Apply one-to-one correspondence and number patterns to count up and count back and to compare values of whole numbers and values of money.	Apply number patterns to count and compare values of whole numbers, fractions and decimals.	Model and compare values of integers and rational numbers .	Model and compare values of irrational and complex numbers.
B. Represent numbers in equivalent forms	Represent equivalent forms of the same number through the use of concrete objects, drawings, word names and symbols.	Use number theory concepts and models to represent or rename whole numbers, fractions and decimals.	Represent and use numbers in equivalent forms (e.g., integers , fractions, decimals, percents, exponents , scientific notation , square roots, absolute values)	Use factoring to create equivalent forms of polynomials.
C. Concepts of Numbers and Relationships	Use drawings, diagrams or models to show the concept of fraction as part of a whole.	Use models to represent the concept of an integer , fraction, decimal or percent.	Use ratio and proportion to model relationships between quantities.	Intentionally Blank
D. Place Value	Apply place-value concepts and base-ten numeration to order and compare whole numbers.	Apply place-value concepts to order and compare decimals and to express whole numbers and decimals in expanded notation.	Extend place-value concepts to represent large numbers using exponential, scientific and calculator notation.	Use exponential, scientific and calculator notation to represent any rational number .
E. Number Theory	Apply number patterns even and odd, factors and multiples to represent numbers in various ways.	Develop and apply number theory concepts (e.g., primes , factors , multiples , composites) to represent numbers in various ways.	Apply concepts of prime and composite numbers to calculate GCFs (Greatest Common Factor) and LCMs (Least Common Multiple) of numbers.	Apply the concepts of prime and composite polynomials to determine GCFs (Greatest Common Factor) and LCMs (Least Common Multiple) of polynomials.
F. Concepts and Applications of Operations	Understand the concepts of addition and subtraction and use the inverse relationships between addition and subtraction to determine unknown quantities in equations .	Understand the concepts of multiplication and division and use the inverse relationships between multiplication and division, to determine unknown quantities in equations .	Understand the concepts of exponents and roots and use the inverse relationships between exponents and roots to determine unknown quantities in equations .	Understand the concepts of exponential and logarithmic forms and use the inverse relationships between exponential and logarithmic expression to determine unknown quantities in equations .

2.2. Computation and Estimation

Strand	2.2.3. GRADE 3	2.2.5. GRADE 5	2.2.8. GRADE 8	2.2.11. GRADE 11
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>				
A. Fluency in Basic Facts	Develop fluency in the use of basic facts for the four operations.	Intentionally Blank	Intentionally Blank	Intentionally Blank
B. Computation	Add and subtract single- and double-digit numbers with regrouping and triple-digit number, without regrouping including problems with money.	Multiply and divide single- and double-digit numbers; add and subtract fractions and mixed numbers; add, subtract, multiply and divide decimals.	Add, subtract, multiply and divide different kinds and forms of rational numbers including integers , decimal fractions, percents and proper and improper fractions.	Intentionally Blank
C. Evaluate Numerical Expressions	Intentionally Blank	Intentionally Blank	Use the order of operations to evaluate numerical expressions .	Evaluate numerical expressions that include the four basic operations and operations of powers and roots, reciprocals , opposites and absolute values .
D. Numerical Estimation	Estimate values, sums and differences of quantities and conclude the reasonableness of those estimates.	Estimate results from calculations with basic operations of whole numbers and decimals and check the reasonableness of those estimates.	Estimate the values of irrational numbers and the results from calculations with basic operations of fractions and percents and check the reasonableness of those estimates.	Intentionally Blank

2.3. Measurement and Estimation

Strand	2.3.3. GRADE 3	2.3.5. GRADE 5	2.3.8. GRADE 8	2.3.11. GRADE 11
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>				
A. Concept of Measurement	Demonstrate an understanding of measurable characteristics and the need to quantify those characteristics.	Use concrete objects to demonstrate the meaning of measurement quantities (e.g., perimeter, area, weight, capacity).	Intentionally Blank	Intentionally Blank
B. Units and Tools of Measurement	Identify a measurable characteristic of an object, select an appropriate standard or non-standard unit of measure and tool, and determine the measurement to a specified level of accuracy.	Select and use appropriate instruments and units for measuring quantities to a specified level of accuracy.	Develop strategies for determining areas and volumes of compound shapes and solids.	Intentionally Blank
C. Calculations	Tell time on an analog and digital clock, identify times of day and night as a.m. and p.m., and calculate elapsed time.	Calculate perimeter and area, and sums and differences of measurements.	Calculate volume, surface area, and degrees of angles; calculate circumference and area of circles, and use a measurement formula to solve for a missing quantity.	Use properties of geometric figures and measurement formulas to solve for a missing quantity (e.g., the measure of a specific angle created by parallel lines and a transversal).
D. Conversions	Identify equivalent measurements within the same system.	Perform basic conversions within a system.	Perform conversions within the metric system and within the customary system including scale measurements, between units of time and between units of temperature.	Intentionally Blank
E. Relations	Intentionally Blank	Intentionally Blank	Describe how a change in linear dimension of an object affects its perimeter, area and volume.	Describe how a change in the value of one variable in a formula affects the value of the measurement.
F. Measurement Estimation	Estimate and verify measurements of length, area, weight, and capacity.	Estimate and verify measurements of length, perimeter, area, volume, capacity, temperature, time, weight and angles.	Estimate and verify measurements of rate and mass.	

2.4. Mathematical Reasoning and Connections

Strand	2.4.3. GRADE 3	2.4.5. GRADE 5	2.4.8. GRADE 8	2.4.11. GRADE 11
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>				
A. Reasoning	Use models and number facts to draw conclusions and explain reasons for conclusions.	Use models, number facts, properties and relationships to draw conclusions and explain reasons for conclusions.	Draw inductive and deductive conclusions within mathematical contexts.	Write formal proofs (direct proofs, indirect proofs/ proofs by contradiction , use of counter-examples, truth tables, etc.) to validate conjectures or arguments.
B. Connections	Interpret statements made with precise language of logic (e.g., “all”, “or”, “every”, “none”, “some”, “or”, “many”).	Use if...then statements to express conditional relationships.	Use if...then statements to construct simple valid arguments .	Use statements, converses, inverses and contra positives to construct valid arguments or to validate arguments.

2.5. Mathematical Problem Solving and Communication

Strand	2.5.3. GRADE 3	2.5.5. GRADE 5	2.5.8. GRADE 8	2.5.11. GRADE 11
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>				
A. Problem Solving	Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense and explain how the problem was solved in grade appropriate contexts.	Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense and explain how the problem was solved in grade appropriate contexts.	Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, apply estimation skills as appropriate, check whether the plan makes sense and explain how the problem was solved in grade appropriate contexts.	Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense and explain how the problem was solved in grade appropriate contexts.
B. Communication	Use appropriate mathematical vocabulary when explaining how to solve a problem.	Use appropriate mathematical terms, vocabulary, language, symbols and graphs to explain clearly and logically solutions to problems.	Use precise mathematical language, notation and representations, including numerical tables and equations , simple algebraic equations and formulas, charts, graphs and diagrams to explain and interpret results.	Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results.

2.6. Statistics and Data Analysis				
Strand	2.6.3. GRADE 3	2.6.5. GRADE 5	2.6.8. GRADE 8	2.6.11. GRADE 11
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>				
A. Collection of Data	Gather data from surveys and observations within the classroom or homes.	Gather data from surveys and observations from sources outside the classroom or home.	Understand and apply sampling techniques to gather data including simple random sampling and convenience sampling.	Design and conduct an experiment using random sampling.
B. Organization and Display of Data	Organize and display data using pictures, tallies, charts, bar graphs and pictographs.	Use pictures, tallies, tables, charts, bar graphs, line graphs, diagrams, and graphs to organize, display, and analyze data. (Standard wording was rearranged to insert the word <i>analyze</i> in a logical place.)	Organize and display one- variable data using appropriate data display, such as stem and leaf and box-and whisker plots , and two variable data with scatterplots .	Intentionally Blank
C. Numerical Summaries	Describe data displayed in a diagram (e.g., Venn) a graph or a table.	Calculate mean and range , identify the median and the mode of a set of data, and use these quantities to describe the data.	Calculate quartiles for one- variable data and describe the correlation coefficient for two- variable data displayed in a scatterplot .	Select or calculate the appropriate measure of central tendency , calculate and apply the interquartile range for on- variable data, and construct a line of best fit and calculate its equation for two- variable data.
D. Statistical Comparisons	Analyze data shown in tables, charts, diagrams, and graphs; compare the data from two categories displayed in a graph and compare representations of a set of data in different graphs.	Compare data using multiple categories displayed in a graph.	Compare data sets graphically using double-bar and double-line graphs and numerically using mean, median, mode, range and quartiles .	Intentionally Blank
E. Interpretation of Data	Determine the reasonableness of a statement based on a comparison to data displayed in a graph.	Determine the reasonableness of a statement based on a comparison to data displayed in a graph and summarized by numerical measures.	Determine the effect of extreme values on numerical summaries and calculate estimates based on survey results or graphs.	Make predictions based on lines of best fit or draw conclusions on the value of a variable in a population based on the results of a sample.

2.7. Probability and Predictions

Strand	2.7.3. GRADE 3	2.7.5. GRADE 5	2.7.8. GRADE 8	2.7.11. GRADE 11
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>				
A. Calculate Probabilities	Determine the chance of an event occurring by performing simulations with concrete devices (e.g., dice, spinner).	Predict and calculate the likelihood of simple events .	Calculate the probability of an event involving “and”, “or” or “not”.	Use probability to predict the likelihood of an outcome in an experiment.
B. Prediction of Outcomes	Determine whether different outcomes of the same event are equally likely or not equally likely.	Predict and determine why some outcomes of a particular event are certain, more likely, less likely, equally likely or impossible.	Intentionally Blank	Intentionally Blank
C. Representations of Probabilities	Write the likelihood of an event as a fraction.	Express probabilities as fractions and/or decimals.	Determine the number of combinations and permutations for an event.	Compare odds and probability .
D. Display Sample Spaces	List or graph the possible results of an experiment.	List all possible combinations and arrangements of outcomes of an experiment (i.e. tree diagrams , matrices , etc.)	Intentionally Blank	Intentionally Blank
E. Compare Theoretical and Experimental Probabilities	Determine that there can be a difference between predicted and actual outcomes.	Compare predictions based on theoretical probability and experimental results.	Find the experimental or theoretical probability of the outcomes of a simple or compound event.	Use probability to make judgments about the likelihood of various outcomes.

2.8. Algebra and Functions				
Strand	2.8.3. GRADE 3	2.8.5. GRADE 5	2.8.8. GRADE 8	2.8.11. GRADE 11
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>				
A. Algebraic Properties	Use the concept of equality and concrete objects to demonstrate understanding of commutative, associative and identity properties.	Use the concept of equality to demonstrate understanding of the distributive property.	Use the concept of equality to demonstrate understanding of the inverse properties of numbers and the addition and multiplication properties of equality.	Intentionally Blank
B. Algebraic Manipulations	Use concrete objects and trial and error to solve number sentences (equations and inequalities).	Select and use strategies, including concrete objects, to solve number sentences (equations and inequalities) and explain the method of solution.	Evaluate and simplify algebraic expressions and solve and graph linear equations and inequalities .	Evaluate and simplify algebraic expressions and solve and graph linear, quadratic, exponential and logarithmic equations and inequalities , and solve and graph systems of equations and inequalities .
C. Patterns	Recognize, describe, extend, create and replicate a variety of patterns including attribute, activity, number and geometric patterns .	Recognize, describe, extend, create, replicate and form a rule for a variety of patterns , sequences , and relationships verbally, numerically, symbolically and graphically.	Find the missing elements and recognize, describe and extend patterns to include linear , exponential , and simple quadratic equations.	Recognize, describe and generalize pattern using sequences and series to predict long-term outcomes.
D. Functions	Use a rule to find a missing value, and determine a rule for a given pattern.	Determine a functional rule from a table or graph.	Create a table or graph from a function a l rule.	Demonstrate an understanding and apply properties of functions (domain , range, inverses) and, characteristics of families of functions (linear , polynomial, rational, trigonometric, exponential, logarithmic).
E. Modeling	Use concrete objects or combinations of symbols and numbers to represent expressions , equations and inequalities that model mathematical situations.	Use concrete objects and combinations of symbols and numbers to create expressions , equation and inequalities that model mathematical situations.	Use combinations of symbols and numbers to create expressions , equations in one or two variables and inequalities in one variable that model problem situations.	Use combinations of symbols and numbers to create expressions , equations and inequalities in two or more variables , systems of equations and inequalities , and functional relationships, that model problem situations.
F. Interpret Results of Modeling	Describe data represented in a table, chart or number sentence and/or create a story that matches that data.	Describe data represented in equations , inequalities , tables or graphs and/or create a story that matches that data.	Interpret the results of solving equations in one or two variables and inequalities in one variable in the context of the situation that motivated the model.	Interpret the results of solving equations , inequalities , systems of equations and inequalities in the context of the situation that motivated the model.

2.9 Geometry				
Strand	2.9.3. GRADE 3	2.9.5. GRADE 5	2.9.8. GRADE 8	2.9.11. GRADE 11
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>				
A. Definitions, Properties and Relations	Name, describe and draw/build 2- and 3-dimensional shapes.	Identify, describe, and define 1-, 2-, and 3-dimensional shapes and their related parts, and classify and compare 2- and 3- dimensional shapes on the basis of their properties.	Name, describe and apply geometric relations for 1- dimensional shapes and 2- dimensional shapes and 3- dimensional solids.	Create justifications for arguments related to geometric relations.
B. Transformations and Symmetry	Identify and draw lines of symmetry .	Predict and describe the result of a translation (slide), rotation (turn), or reflection (flip) of a 2- dimensional shape.	Predict and describe the result of a translation (slide), rotation (turn), or reflection (flip) of a 3- dimensional shape.	Use arguments based on transformations to establish congruence or similarity of 2- dimensional shapes
C. Coordinate Geometry	Identify locations of points with whole number coordinates on a number line or on a 2- dimensional coordinate system .	Identify location of points with fractional or decimal coordinates on a number line or on a 2- dimensional coordinate system .	Plot ordered pairs and 2-dimensional shapes that satisfy given conditions on a 2-dimensional coordinate system .	Use techniques from coordinate geometry to establish properties of lines, shapes and solids.

2.10 Trigonometry				
Strand	2.10.3. GRADE 3	2.10.5. GRADE 5	2.10.8. GRADE 8	2.10.11. GRADE 11
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>				
A. Right Triangle Concepts and Applications	Identify right angles in the environment.	Identify and compare parts of right triangles, including right angles, acute angles, hypotenuses and legs.	Compute measures of sides and angles using proportions , the Pythagorean Theorem and right triangle relationships.	Identify, create and solve practical problems involving right triangles using the trigonometric functions and the Pythagorean Theorem.
B. Trigonometric Functions	Intentionally Blank	Intentionally Blank	Intentionally Blank	Graph periodic and circular functions; describe properties of the graphs.

2.11 Concepts of Calculus				
Strand	2.11.3. GRADE 3	2.11.5. GRADE 5	2.11.8. GRADE 8	2.11.11. GRADE 11
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>				
A. Extreme Values	Identify whole number quantities and measurements from least to most and greatest value.	Make comparisons of numbers (e.g., more, less, same, least, most, greater than, less than).	Analyze graphs of related quantities for minimum and maximum values and justify the findings.	Determine and interpret maximum and minimum values of a function over a specified interval.
B. Rates	Intentionally Blank	Describe the relationship between rates of change and another variable (e.g. time, temperature).	Describe the concept of unit rate, ratio and slope in the context of rate of change.	Analyze and interpret rates of growth/decay.
C. Accumulation of Areas and Volumes	Intentionally Blank	Estimate areas and volumes of shapes and solids as the sums of areas of tiles and volumes of cubes.	Intentionally Blank	Estimate areas under curves using sums of areas.

VI. GLOSSARY

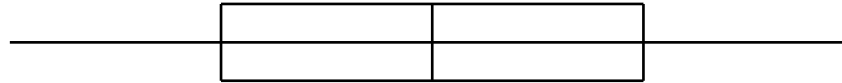
Absolute value: A number's distance from zero on a number line. The absolute value of 2 is equal to the absolute value of -2.

Algorithm: A method of performing an arithmetic operation.

Analog time: Time displayed on a timepiece having hour and minute hands.

Array: Arrangement of a series of items according to the values of the items (e.g., largest to smallest).

Box-and-whisker plot: A graphic method for showing a summary of data using median, **quartiles** and extremes of data.



Central Tendency: The degree of clustering of the values of a statistical distribution that is usually measured by the arithmetic mean, mode, or median.

Combination: A subset of the elements in a given set, without regard to the order in which those elements are arranged.

Composite number: Any positive integer exactly divisible by one or more positive integers other than itself and 1.

Congruent: Having the same shape and the same size.

Conjecture: A statement believed to be true but not proved.

Coordinate system:	A method of locating points in the plane or in space by means of numbers. A point in the plane is located by its distances from both a horizontal and a vertical line called the axes. The horizontal line is called the x-axis. The vertical line is called the y-axis. The pairs of numbers are called ordered pairs. The first number, called the x-coordinate, designates the distance along the horizontal axis. The second number, called the y-coordinate, designates the distance along the vertical axis. The point at which the two axes intersect has the coordinates (0,0) and is called the origin.
Correlation:	A measure of the mutual relationship between two variables .
Customary system:	A system of weights and measures frequently used in the United States. The basic unit of weight is the pound; the basic unit of capacity is the quart.
Deductive reasoning:	The process of reasoning from statements accepted as true to reach a conclusion.
Direct variation:	Two variables are so related that their ratio remains constant.
Domain:	The set of all possible values for the unknown in an open sentence.
Equation:	A statement of equality between two mathematical expressions (e.g., $X + 5 = Y - 2$).
Equivalent forms:	Different forms of numbers that name the same number (e.g., fraction, decimal, percent as $\frac{1}{2}$, .5, 50%).

Expanded notation:	Involves writing the number in expanded form to show the value of each digit (e.g., $15,629 = 10,000 + 5,000 + 600 + 20 + 9$).
Exponential function:	A function whose general equation is $y = a \times b^x$ or $y = a \times b^{kx}$, where a, b and k stand for constants.
Exponent:	A numeral used to tell how many times a number or variable is used as a factor (e.g., a^2 , 2^n , y^x).
Expression:	A mathematical phrase that can include operations, numerals and variables . In algebraic terms: $2l + 3x$; in numeric terms: $13.4 - 4.7$.
Factor:	The number or variable multiplied in a multiplication expression.
Factorial:	The expression $n!$ (n factorial) is the product of all the numbers from 1 to n for any positive integer n.
Function:	A relation in which each value of an independent variable is associated with a unique value of the dependent value.
Geoboard:	A board with pegs aligned in grid fashion that permits rubber bands to be wrapped around pegs to form geometric figures.
Graphing calculator:	A calculator that will store and draw the graphs of several functions at once.
Independent events:	Events such that the outcome of the first event has no effect on the probabilities of the outcome of the second event (e.g., two tosses of the same coin are independent events).

Inductive reasoning:	Generalizations made from particular observations in a common occurrence.
Inequality:	A mathematical sentence that contains a symbol (e.g., $>$, $<$, \geq , \leq or \neq) in which the terms on either side of the symbol are unequal (e.g., $x < y$, $7 > 3$, $n \geq 4$).
Infinite:	Has no end or goes on forever.
Integer:	A number that is a positive whole number, a negative whole number or zero.
Inverse:	A new conditional formed by negating both the antecedent and the consequent of a conditional.
Inverse operations:	Operations that undo each other (e.g., addition and subtraction are inverse operations; multiplication and division are inverse operations).
Inverse variation:	When the ratio of one variable to the reciprocal of the other is constant, one of them is said to vary inversely as the other.
Irrational number:	A number that cannot be written as a simple fraction. It is an infinite and non-repeating decimal.
Limit:	A number to which the terms of a sequence get closer so that beyond a certain term all terms are as close as desired to that number.

Line of best fit:	The line that fits a set of data points with the smallest value for the sum of the squares of the errors (vertical distances) from the data points to the line; the regression line.
Linear function:	A function whose general equation is $y = mx + b$, where m and b stand for constants and $m \neq 0$.
Linear measurement:	Measurement in a straight line.
Logarithm:	The exponent indicating the power to which a fixed number, the base, must be raised to produce a given number. For example, if $n^x = a$, the logarithm of a , with n as the base, is x ; symbolically, $\log_n a = x$. If the base is 10, the log of 100 is 2.
Manipulatives:	Materials that allow students to explore mathematical concepts in a concrete mode.
Mathematical model:	A representation in the mathematical world of some phenomenon in the real world. It frequently consists of a function or relation specifying how two variables are related.
Matrix:	A rectangular array of numbers representing such things as the coefficients in a system of equations arranged in rows and columns.
Maximum:	The greatest number in a set of data.
Mean:	The sum of the set of numbers divided by n , the number of numbers in the set.

Median:	The number that lies in the middle when a set of numbers is arranged in order. If there are two middle values, the median is the mean of these values.
Metric system:	A system of measurement used throughout the world based on factors of 10. It includes measures of length, weight and capacity.
Minimum:	The least number in a set of data.
Missing addend:	A member of an addition number sentence in which that term is missing (e.g., $5 + \underline{\quad} = 8$).
Mode:	The number(s) that occurs most often in a set of numbers (e.g., in the set 1, 2, 3, 3, 5, 8; the mode is 3).
Multiple:	A number that is the product of a given integer and another integer (e.g., 6 and 9 are multiples of 3).
Normal curve:	A graphical plot of a mathematical function (frequency distribution) which is unimodal and symmetrical.
<i>One-to-one</i> correspondence:	When one and only one element of a second set is assigned to an element of a first set, all elements of the second set are assigned, and every element of the first set has an assignment, the mapping is called one-to-one (e.g., in the set Bill Clinton, George Bush, Ronald Reagan, Jimmy Carter, Hillary Clinton, Barbara Bush, Nancy Reagan and Rosalynn Carter, there is a one-to-one correspondence between the pairs).

Open sentence: A statement that contains at least one unknown. It becomes true or false when a quantity is substituted for the unknown (e.g., $x + 5 = 9$, $y - 2 = 7$).

Order of operations: Rules for evaluating an expression: work first within parentheses; then calculate all powers, from left to right; then do multiplications or divisions, from left to right; then do additions and subtractions, from left to right.

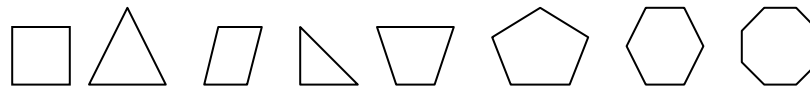
Patterns: Regularities in situations such as those in nature, events, shapes, designs and sets of numbers (e.g., spirals on pineapples, geometric designs in quilts, the number sequence 3, 6, 9, 12,...).

Permutation: An arrangement of a given number of objects from a given set in which the order of the objects is significant.

Perpendicular lines: Two lines that intersect to form right angles (e.g., \perp , \lrcorner , \Uparrow).

Plotting points: Locating points by means of coordinates, or a curve by plotted points, representing an equation by means of a curve so constructed.

Polygon: A union of segments connected end to end, such that each segment intersects exactly two others at its endpoints.



Powers: A number expressed using an exponent. The number 5^3 is read five to the third power or five cubed.

Prime: An integer greater than one whose only positive factors are 1 and itself (e.g., 2, 3, 5, 7, 11, 13, 17, and 19).

Probability:	A number from 0 to 1 that indicates how likely something is to happen.
Problem-solving:	Finding ways to reach a goal when no routine path is apparent.
Proof by contradiction:	A proof in which, if s is to be proven, one reasons from not-s until a contradiction is deduced; from this it is concluded that not-s is false, which means that s is true.
Proportion:	An equation of the form $\frac{a}{b} = \frac{c}{d}$ that states that the two ratios are equivalent.
Quadrilateral:	A four-sided polygon.
Quartiles:	The three values that divide an ordered set into four subsets of approximately equal size. The second quartile is the median.
Radian:	A unit of angular measure equal to $\frac{1}{2\pi}$ of a complete revolution.
Range (1) :	The difference between the greatest number and the least number in a set of data.
Range (2) :	The set of output values for a function.
Rate of change:	The limit of the ratio of an increment of the function value at the point to that of the independent variable as the increment of the variable approaches zero.

Ratio:	A comparison of two numbers by division.
Rational numbers:	Any number that can be written in the form $\frac{a}{b}$ where a is any integer and b is any integer except zero.
Real numbers:	The set consisting of all rational numbers and all irrational numbers.
Reasonableness:	Quality of a solution such that it is not extreme or excessive.
Reciprocal:	The fractional number that results from dividing one by the number.
Rectangular prism:	A three-dimensional figure whose sides are all rectangles; a box.
Reflection:	A transformation that produces the mirror image of a geometric figure.
Regression:	The line that represents the least deviation from the points in a scatter plot of data.
Regular polygon:	A polygon in which all sides have the same measure and all angles have the same measure.
Relation:	A set of ordered pairs.
Reliability:	The extent to which a measuring procedure yields the same results on repeated trials.
Repeated addition:	A model for multiplication (e.g., $2 + 2 + 2 = 3 \times 2$).

Rotation:	A transformation that maps every point in the plane to its image by rotating the plane around a fixed point or line.
Scatterplot:	A graph of plotted points that show the relationship between two sets of data.
Scientific calculator:	A calculator that represents very large or very small numbers in scientific notation with the powering, factorial, square root, negative and reciprocal keys.
Scientific notation:	A way in writing a number of terms of an integer power of 10 multiplied by a number greater than or equal to 1 and less than 10.
Sequence:	A set of ordered quantities (e.g., positive integers).
Series:	The indicated sum of the terms of a sequence.
Similarity:	Having the same shape but not necessarily the same size.
Simple event:	An event whose probability can be obtained from consideration of a single occurrence (e.g., the tossing of a coin is a simple event).
Simulation:	Modeling a real event without actually observing the event.
Slope:	The slope of a line is the ratio of the change in y to the corresponding change in x; the constant m in the linear function equation; rise/run.

Standard deviation:	The square root of the variance.
Stem-and-leaf plot:	<p>A frequency distribution made by arranging data (e.g., student scores on a test were 98, 96, 85, 93, 83, 87, 85, 87, 93, 75, 77 and 83). This data is displayed in a stem-and-leaf plot below.</p> <pre> 9 8, 6, 3, 3 8 7, 7, 5, 5, 3, </pre>
Systems of equations:	Two or more equations that are conditions imposed simultaneously on all the variables , but may or may not have common solutions (e.g., $x + y = 2$, and $3x + 2y = 5$).
Symmetry:	A line of symmetry separates a figure into two congruent halves, each of which is a reflection of the other (e.g., \emptyset , the line through the center of the circle divides it into congruent halves).
t-test:	A statistical test done to test the difference of means of two samples.
Tessellation:	A repetitive pattern of polygons that covers an area with no holes and no overlaps (e.g., floor tiles).
Transformation:	An operation on a geometric figure by which each point gives rise to a unique image.
Translation:	A transformation that moves a geometric figure by sliding each of the points the same distance in the same direction.
Tree diagram:	A diagram used to show the total number of possible outcomes in a probability experiment.

Trigonometric functions: A function (e.g., sine, cosine, tangent, cotangent, secant, cosecant) whose independent **variable** is an angle measure, usually in degrees or radians.

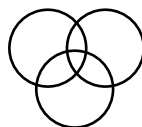
Valid argument: An argument with the property that no matter what statements are substituted in the premises, the truth value of the form is true. If the premises are true, then the conclusion is true.

Variable: A symbol used to stand for any one of a given set of numbers or other objects (e.g., in the equation $y = x + 5$, y and x are **variables**).

Variance: In a data set, the sum of the squared deviations divided by one less than the number of elements in the set (sample variance s^2) or by the number of elements in the set (population variance σ^2).

Vector: A quantity that has both magnitude and direction (e.g., physical quantities such as velocity and force).

Venn diagram: A display that pictures unions and intersections of sets.



Volume: The amount of space enclosed in a space (3-dimensional) figure, measured in cubic units.

Y-intercept: The y-intercept of a line is the y-coordinate of the point at which the graph of an equation crosses the y-axis.

π : pi, the ratio of the circumference of a circle to its diameter: 3.1415926535.